



## Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact [support@jstor.org](mailto:support@jstor.org).

$$\frac{d\rho}{d\theta} = \pm \frac{a\rho}{\sqrt{(b^2 - a^2)}}, \text{ or } \kappa\rho = e^{\pm [a/\sqrt{(b^2 - a^2)}]\theta}.$$

The cylinder has therefore for its right section a logarithmic spiral or rather two arranged in opposite directions.

3. This curious result may be stated in the following way. If an elliptical hole be cut out of a sheet of paper and the sheet wrapped about a sphere the diameter of which equals the minor axis of the ellipse the sheet wraps up into a cylinder a cross section of which is a logarithmic spiral. A model illustrating this relation between the ellipse and the spiral is easily constructed.

BERKELEY, April, 1904.

---

## DEPARTMENTS.

---

### SOLUTIONS OF PROBLEMS.

---

#### ALGEBRA.

---

200. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, O.

No matter what value  $x$  be given, the *numerical* value of the expression  $(x+2)/(2x^2+3x+6)$  can never exceed  $\frac{1}{3}$ .

II. Solution by G. W. GREENWOOD, M.A., Professor of Mathematics and Astronomy in McKendree College, Lebanon, Ill.

We know that  $y+1/y \leq 2$ .

$$\therefore z/2 + 2/z \leq 2. \quad 2z + 8/z \leq 8. \quad 2z + 8/z - 5 \leq 3. \quad \frac{2z^2 - 5z + 8}{z} \leq 3.$$

Now put  $z=x+2$  and we get

$$\frac{2x^2+3x+6}{x+2} \leq 3; \text{ i. e., } \frac{x+2}{2x^2+3x+6} \geq \frac{1}{3}.$$

202. Proposed by G. B. M. ZERR, A. M., Ph. D., Parsons. W. Va.

Express in the form of radicals the roots of the equation

$$x^9 + 9mx^7 + 27m^2x^5 + 30m^3x^3 + 9mx + 2r = 0.$$

I. Solution by A. H. HOLMES, Brunswick, Maine.

Writing  $x^3 + 3mx = y$  the equation becomes

$$y^3 + 3m^3y + 2r = 0 \dots\dots\dots (1).$$

Let  $s_1 = [-r + \sqrt{(m^9 + r^2)}]^{\frac{1}{3}}$ ,  $s_2 = [-r - \sqrt{(m^9 + r^2)}]^{\frac{1}{3}}$ , and the roots of (1) become  $y_1 = s_1 + s_2$ ,  $y_2 = \frac{1}{2}(s_1 + s_2) + \frac{1}{2}\sqrt{(-3)(s_1 - s_2)}$ ,  $y_3 = -\frac{1}{2}(s_1 + s_2) - \frac{1}{2}\sqrt{(-3)(s_1 - s_2)}$ .

$\therefore x^3 + 3mx = y_1$  or  $y_2$  or  $y_3$  ..... (2).

The nine roots of the three cubics (2) give the required solution. We let

$$s_3 = \left(-\frac{y_1}{2} + \sqrt{(m^3 + \frac{1}{4}y_1^2)}\right)^{\frac{1}{3}}, \quad s_4 = \left(-\frac{y_1}{2} - \sqrt{(m^3 + \frac{1}{4}y_1^2)}\right)^{\frac{1}{3}}$$

and we denote the corresponding radicals when  $y_1$  is replaced by  $y_2$ , by  $s_5, s_6$ ; and when  $y_1$  is replaced by  $y_3$  by  $s_7, s_8$ . The nine roots of the given equation are:

$$\begin{aligned} x_1 &= s_3 + s_4, & x_4 &= s_5 + s_6, \\ x_2 &= -\frac{1}{2}(s_3 + s_4) + \frac{1}{2}\sqrt{(-3)(s_3 - s_4)}, & x_5 &= -\frac{1}{2}(s_5 + s_6) + \frac{1}{2}\sqrt{(-3)(s_5 - s_6)}, \\ x_3 &= -\frac{1}{2}(s_3 + s_4) - \frac{1}{2}\sqrt{(-3)(s_3 - s_4)}, & x_6 &= -\frac{1}{2}(s_5 + s_6) - \frac{1}{2}\sqrt{(-3)(s_5 - s_6)}, \\ & & x_7 &= s_7 + s_8, \\ & & x_8 &= -\frac{1}{2}(s_7 + s_8) + \frac{1}{2}\sqrt{(-3)(s_7 - s_8)}, \\ & & x_9 &= -\frac{1}{2}(s_7 + s_8) - \frac{1}{2}\sqrt{(-3)(s_7 - s_8)}. \end{aligned}$$

Similarly solved by Elmer Schuyler, and J. Scheffer.

## II. Solution by the PROPOSER.

Let  $x = y + z$ . Then  $x^9 + 9mx^7 + 27m^2x^5 + 30m^3x^3 + 9m^4x + 2r = 0$ , becomes  
 $y^9 + z^9 + (yz + m)[9(y^7 + z^7) + (36yz + 27m)(y^5 + z^5) + (84y^2z^2 + 105myz + 30m^2)(y^3 + z^3) + (126y^3z^3 + 189my^2z^2 + 81m^2yz + 9m^3)(y + z) + 2r = 0$ .  
 $\therefore y^9 + z^9 = -2r$  and  $yz = -m$ .

$\therefore y^{18} + 2ry^9 = m^9$ .  $\therefore y^9 = -r \pm \sqrt{(r^2 + m^9)}$ .

Let  $a^9 = -r + \sqrt{(r^2 + m^9)}$ ,  $b^9 = -r - \sqrt{(r^2 + m^9)}$ .

Let  $w = \frac{-1 + \sqrt{(-3)}}{2} = \beta^3$ .

Then the nine roots are  $a + b$ ,  $\beta a + \beta^8 b$ ,  $\beta^2 a + \beta^7 b$ ,  $\beta^3 a + \beta^6 b$ ,  $\beta^4 a + \beta^5 b$ ,  $\beta^5 a + \beta^4 b$ ,  $\beta^6 a + \beta^3 b$ ,  $\beta^7 a + \beta^2 b$ ,  $\beta^8 a + \beta b$ .

Also solved by L. E. Newcomb, and F. D. Posey.

203. Proposed by L. E. NEWCOMB, Los Gatos, Cal.

The sum of a certain pair of roots of  $x^4 + ax^3 + \left(\frac{2b}{a} + \frac{a^2}{4}\right)x^2 + bx + c = 0$  is equal to the sum of the remaining pair.

## I. Solution by J. SCHEFFER, Hagerstown, Md.

The problem reduces to the problem of finding the value of  $m$  in the equation  $a^4 + ax^3 + mx^2 + bx + c = 0$ , subject to the condition that the sum of two roots equals the sum of the two others. Denoting the roots by  $\alpha, \beta, \gamma, \delta$ , we have  $\alpha + \beta = \gamma + \delta = -\frac{1}{2}a$ . Since  $\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta = -b$ , we have by factoring,

$$\alpha\beta(\gamma + \delta) + (a + \beta)\gamma\delta = -b, \text{ or } (a\beta + \gamma\delta)\frac{1}{2}a = b.$$